Regulating Photon Mass in Classical 5D Gauge Theory

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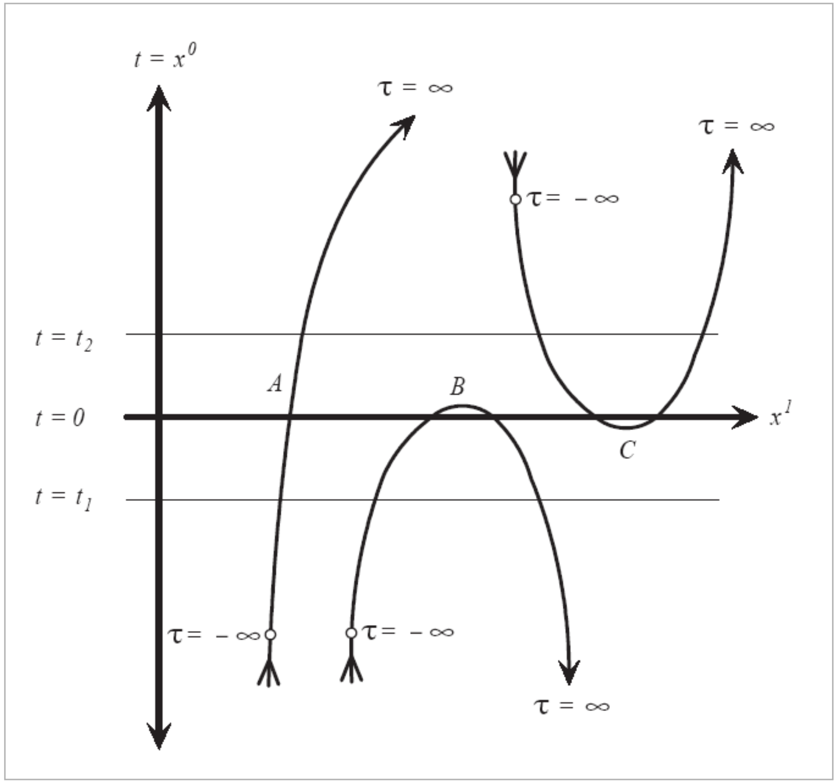
Stueckelberg's Model of Pair Creation/Annihilation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = eF^{\mu\nu}g_{\nu\rho}\frac{dx^\rho}{d\tau} + K^\mu$$



$\text{diag}(-,+,+,+) \qquad \mu,\nu,\rho = 0,\cdots,3$

$$\Gamma^{\mu\lambda\nu} = -\frac{1}{2}(\partial^\nu g^{\lambda\mu} + \partial^\lambda g^{\mu\nu} - \partial^\mu g^{\lambda\nu})$$

$B^{\mu\nu}(x)$, a field strength tensor

$K^\mu(x)$, a field strength four-vector

$$m^2 = -g_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau}$$

$$K^\mu = 0 \quad \rightarrow \quad m^2 = \text{constant}$$

Figure 1: World Lines

A: Usual type, with a unique solution to $t(\tau) = x^0$ for each x^0

B: Annihilation type, with two solutions to $t(\tau) = x^0$ for $x^0 \ll 0$ and no solution for $x^0 \gg 0$

C: Creation type, with two solutions to $t(\tau) = x^0$ for $x^0 \gg 0$ and no solution for $x^0 \ll 0$

E. C. G. Stueckelberg, Helv. Phys. Acta **14** (1941) 322

Gauge Symmetries of Stueckelberg's Model

Lagrangian Structure

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{x}_\mu} - \frac{\partial L}{\partial x_\mu} = 0 \qquad L = \frac{1}{2} M \dot{x}^\mu \dot{x}_\mu + e\dot{x}^\mu A_\mu(x)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Classical Hamiltonian

$$\frac{dx^\mu}{d\tau} = \frac{\partial K}{\partial p_\mu} \qquad \frac{dp^\mu}{d\tau} = -\frac{\partial K}{\partial x_\mu} \qquad K = \frac{1}{2M}(p^\mu - eA^\mu)(p_\mu - eA_\mu)$$

Quantum Mechanics

$$i\partial_\tau \psi(x,\tau) = \frac{1}{2M}(p^\mu - eA^\mu)(p_\mu - eA_\mu)\psi(x,\tau)$$

Local Gauge Symmetry

$$\psi(x,\tau) \to \exp\left[ie\Lambda(x)\right]\ \psi(x,\tau)$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda(x)$$

Global Gauge Symmetry

$$\partial_\mu j^\mu + \partial_\tau \rho = 0$$

$$\rho = \left|\,\psi(x,\tau)\,\right|^2 \qquad j^\mu = -\frac{i}{2M}\left\{\psi^*(\partial^\mu - ieA^\mu)\psi - \psi(\partial^\mu + ieA^\mu)\psi^*\right\}$$

$$\rho(x,\tau) \xrightarrow[\tau\to\pm\infty]{} 0 \ \Rightarrow\ \partial_\mu J^\mu(x) = \partial_\mu \int_{-\infty}^{\infty} d\tau\ j^\mu(x,\tau) = 0$$

Enlarged Gauge Symmetry

Locally Gauge Symmetric Quantum Mechanics

$$\psi(x,\tau) \to e^{ie_0\Lambda(x,\tau)}\psi(x,\tau) \qquad a_\mu(x,\tau) \to a_\mu(x,\tau) + \partial_\mu\Lambda(x,\tau)$$

$$a_5(x,\tau) \to a_5(x,\tau) + \partial_\tau\Lambda(x,\tau)$$

$$(i\partial_\tau + e_0 a_5)\psi(x,\tau) = \frac{1}{2M}(p^\mu - e_0 a^\mu)(p_\mu - e_0 a_\mu)\psi(x,\tau)$$

Conventions $x^5 = \tau \qquad \lambda,\mu,\nu = 0,1,2,3 \qquad \alpha,\beta,\gamma = 0,1,2,3,5$

Action

$$S = \int d^4x d\tau \left\{ \psi^*(i\partial_\tau + e_0 a_5)\psi - \frac{1}{2M}\psi^*(p_\mu - e_0 a_\mu)(p^\mu - e_0 a^\mu)\psi - \frac{\lambda}{4} f_{\alpha\beta} f^{\alpha\beta} \right\}$$

Field Equations $\partial_\beta f^{\alpha\beta} = \frac{e_0}{\lambda} j^\alpha = e j^\alpha \qquad \epsilon^{\alpha\beta\gamma\delta\epsilon}\partial_\alpha f_{\beta\gamma} = 0 \qquad f_{\alpha\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha$

3-Vector Field Equations

$$\nabla \cdot \mathbf{e} - \partial_\tau \varepsilon^0 = e j^0 \qquad \nabla \times \mathbf{e} + \partial_0 \mathbf{h} = 0$$

$$\nabla \times \mathbf{h} - \partial_0 \mathbf{e} - \partial_\tau \boldsymbol{\varepsilon} = e\mathbf{j} \qquad \nabla \cdot \mathbf{h} = 0$$

$$\nabla \cdot \boldsymbol{\varepsilon} + \partial_0 \varepsilon^0 = e j^5 \qquad \nabla \times \boldsymbol{\varepsilon} - \sigma\partial_\tau \mathbf{h} = 0$$

$$\nabla \varepsilon^0 + \sigma\partial_\tau \mathbf{e} + \partial_0 \boldsymbol{\varepsilon} = 0$$

$$e_i = f^{0i} \qquad h_i = \varepsilon_{ijk} f^{jk}$$

$$\varepsilon^i = f^{5i} \qquad \varepsilon^0 = f^{50}$$

Classical Off-Shell Electrodynamics

Lorentz Force

$$M\frac{d^2x^\mu}{d\tau^2} = e_0 f^{\mu\alpha}\frac{dx_\alpha}{d\tau} = e_0\left[f^{\mu\nu}\frac{dx_\nu}{d\tau} - \sigma f^{\mu 5}\right]$$

Flat metric

$$g_{\alpha\beta} = \mathrm{diag}(-1,1,1,1,\sigma)$$

Dynamic Mass Conservation

$$\frac{d}{d\tau}\left(-\frac{1}{2}M\dot{x}^2\right) = -M\dot{x}^\mu\ddot{x}_\mu = -\dot{x}^\mu e_0\left(f_{\mu 5} + f_{\mu\nu}\dot{x}^\nu\right) = e_0 f_{5\alpha}\dot{x}^\alpha$$

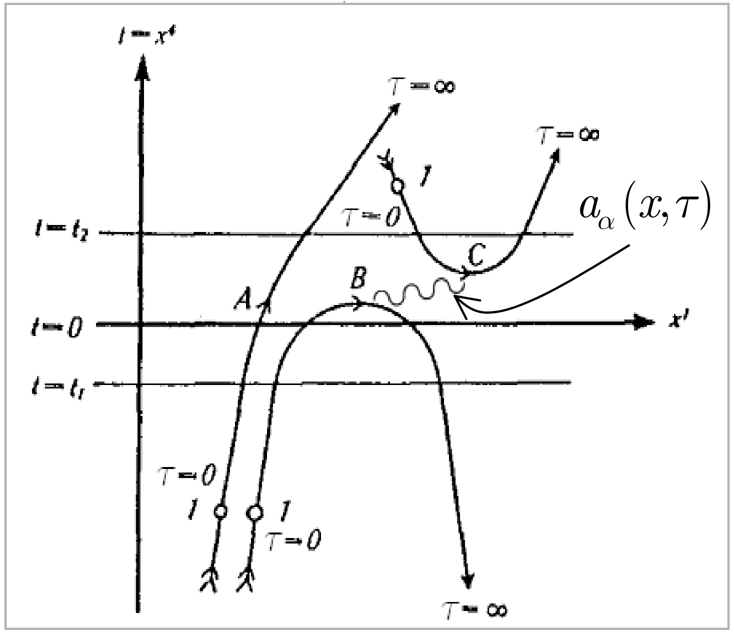
$$f_{5\mu} = 0 \quad \Rightarrow \quad \partial_\tau f^{\mu\nu} = 0$$

Wave Equation

$$\partial_\alpha\partial^\alpha f^{\beta\gamma} = (\partial_\mu\partial^\mu + \sigma\,\partial_\tau^2)f^{\beta\gamma} = -e(\partial^\beta j^\gamma - \partial^\gamma j^\beta)$$

Green's Function

$$G(x,\tau) = -\frac{1}{4\pi}\delta(x^2)\delta(\tau) - \frac{1}{2\pi^2}\frac{\partial}{\partial x^2}\frac{\theta(-\sigma g_{\alpha\beta}x^\alpha x^\beta)}{\sqrt{-\sigma g_{\alpha\beta}x^\alpha x^\beta}}$$



Formal Symmetry

$$\sigma = \begin{cases} 1, & O(4,1) \\ -1, & O(3,2) \end{cases}$$

Connection With Maxwell Theory

Five Dimensional Conserved Current

$$\partial_\alpha j^\alpha = \partial_\mu j^\mu + \partial_\tau j^5 = 0$$

$$j^5 \equiv \rho = | \psi(x,\tau) |^2 \qquad j^\mu = \frac{-i}{2M}\{\psi^*(\partial^\mu - ie_0 a^\mu)\psi - \psi(\partial^\mu + ie_0 a^\mu)\psi^*\}$$

Divergenceless Maxwell Current

$$\partial_\mu j^\mu = -\partial_\tau \rho \neq 0$$

$$\rho \xrightarrow[\tau \to \pm\infty]{} 0 \;\Rightarrow\; \partial_\mu J^\mu(x) = \partial_\mu \int_{-\infty}^{\infty} d\tau \; j^\mu(x,\tau) = 0$$

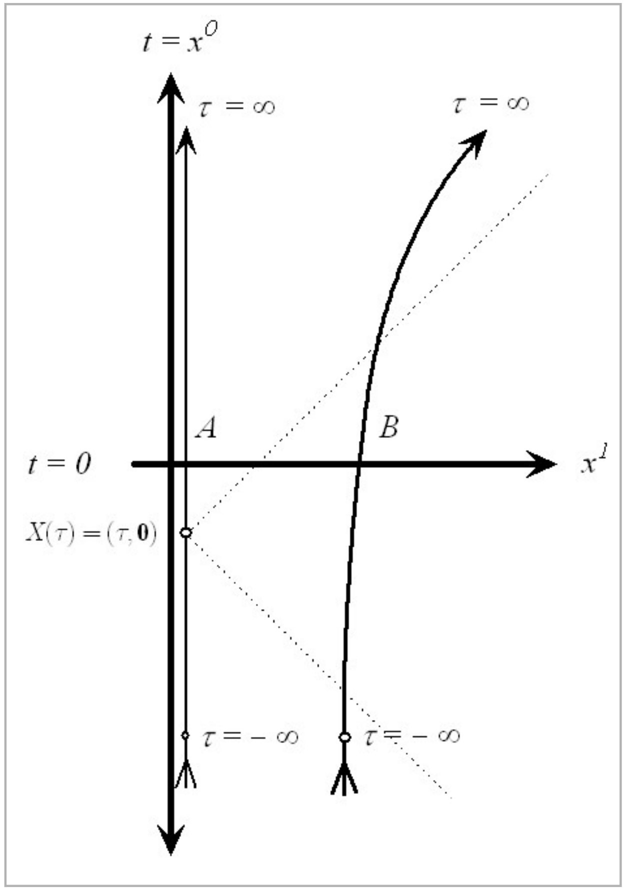
Concatenation

$$f^{5\mu} \xrightarrow[\tau \to \pm\infty]{} 0 \;\Rightarrow\; \left.\begin{matrix} \partial_\beta f^{\alpha\beta} = e j^\alpha \\ \epsilon^{\alpha\beta\gamma\delta\epsilon}\partial_\alpha f_{\beta\gamma} = 0 \end{matrix}\right\} \to \left\{\begin{matrix} \partial_\nu F^{\mu\nu} = e J^\mu \\ \epsilon^{\mu\nu\rho\lambda}\partial_\mu F_{\nu\rho} = 0 \end{matrix}\right.$$

$$F^{\mu\nu}(x) = \int_{-\infty}^{\infty} d\tau \; f^{\mu\nu}(x,\tau)$$

$$A^\mu(x) = \int_{-\infty}^{\infty} d\tau \; a^\mu(x,\tau)$$

Classical Coulomb Scattering Problem



Classical Current $$j^\alpha(x,\tau) = \dot{X}^\alpha(\tau)\ \delta^4(x^\mu - X^\mu(\tau))$$

"Static" Event $$\left(X^0(\tau), \mathbf{X}(\tau)\right) = (\tau, \mathbf{0})$$

Induced Field

$$a^0(x,\tau) = -\frac{e}{4\pi R}\ \frac{1}{2}[\delta(t - R - \tau) + \delta(t + R - \tau)]$$

$$\mathbf{a}(x,\tau) = 0$$

$$a^5(x,\tau) = a^0(x,\tau)$$

Concatenated Current

$$J^\alpha(x) = \int_{-\infty}^{\infty} d\tau\ \dot{X}^\alpha(\tau)\ \delta^4(x^\mu - X^\mu(\tau))$$

Concatenated Potential

$$A^0(x) = -\frac{e}{4\pi R}\int_{-\infty}^{\infty} d\tau\ \ \frac{1}{2}[\delta(t - R - \tau) + \delta(t + R - \tau)] = -\frac{e}{4\pi R}$$

$$\mathbf{A}(x) = 0$$

Mass Regulation Term

Modified Action

$$S_{em-kinetic} = S^0_{em-kinetic} + \frac{\lambda^3}{4} \int d^4x \, d\tau \; (\partial_\tau f^{\alpha\beta}(x,\tau))(\partial_\tau f_{\alpha\beta}(x,\tau))$$

Equivalent Action

$$S_{em-kinetic} = -\frac{\lambda}{4} \int ds \, f^{\alpha\beta}(x,\tau) \, \Phi(\tau - s) f_{\alpha\beta}\Big(x, s\Big)$$

$$\Phi(\tau) = \delta(\tau) - \lambda^2 \delta''(\tau)$$

Modified Field Equations

$$\partial_\beta f^{\alpha\beta}(x,\tau) = e \, j^\alpha_\varphi(x,\tau) = \int_{-\infty}^{\infty} ds \; \varphi(\tau - s) \; j^\alpha(x,s)$$

$$\int_{-\infty}^{\infty} ds \; \Phi(\tau - s) \, \varphi(s - r) = \delta(\tau - r)$$

Spreading Function

$$\Phi(\tau) = \int \frac{d\kappa}{2\pi} \widehat{\Phi}(\kappa) \; e^{-i\kappa\tau} \Rightarrow \qquad \widehat{\Phi}(\kappa) = 1 + (\lambda\kappa)^2$$

$$\varphi(\tau) = \int \frac{d\kappa}{2\pi} \widehat{\varphi}(\kappa) \; e^{-i\kappa\tau} = \int \frac{d\kappa}{2\pi} \frac{1}{1 + (\lambda\kappa)^2} \; e^{-i\kappa\tau} = \frac{1}{2\lambda} e^{-|\tau|/\lambda}$$

Concatenation Invariance

$$\int_{-\infty}^{\infty} d\tau \; \varphi(\tau) = 1 \Rightarrow \int_{-\infty}^{\infty} d\tau \, j^\alpha_\varphi(x,\tau) = \int_{-\infty}^{\infty} ds \; j^\alpha(x,s) = J^\alpha(x)$$

Regulated Classical Scattering Problem

Low Energy Test Event $x^0(\tau) \simeq \tau \quad \Rightarrow \quad |t - \tau \pm R| \simeq R \qquad \frac{dx^0}{d\tau} \simeq 1$

Yukawa Potential $a_\varphi^0(x,\tau) \simeq \frac{1}{2\lambda}\left[-\frac{e}{4\pi R}\, e^{-R/\lambda}\right]\left[\frac{1}{2}\left(\frac{dx^0}{d\tau}+1\right)\right] \simeq \frac{1}{2\lambda}\left[-\frac{e}{4\pi R}\, e^{-R/\lambda}\right]$

Lorentz Force $M\,\frac{d^2\mathbf{x}}{d\tau^2} \simeq 2\lambda\, e\, \nabla\, a_\varphi^0(x,\tau) = \nabla\left[-\frac{e^2}{4\pi R}\, e^{-R/\lambda}\right]$

Observed photon mass : $m_\gamma < 6\times 10^{-17}\, eV$

Regard m_γ as photon mass cut - off $\frac{1}{\lambda} \Rightarrow \lambda \simeq 70$ seconds

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Photon Propagator $\left[\frac{1}{\lambda}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\frac{-i}{k^2+\kappa^2 - i\varepsilon}\right] \times \left(\frac{1}{1+\lambda^2\kappa^2}\right)$

Off-Shell QED is super-renormalizable (finite at two loops)

Distribution of Delay in Event Current

Classical Action

$$L = \dot{x}^\mu p_\mu - K = \frac{1}{2} M \dot{x}^\mu \dot{x}_\mu + e_0 \dot{x}^\alpha a_\alpha = \frac{1}{2} M \dot{x}^\mu \dot{x}_\mu + e_0 \dot{x}^\mu a_\mu + e_0 a_5$$

Not reparameterization invariant, but invariant under $\tau \to \tau - s$

Current as Ensemble Average

$$\begin{aligned} j^\alpha_\varphi(x,\tau) &= \int_{-\infty}^{\infty} ds\ \varphi(\tau - s)\ j^\alpha(x,s) \\ &= \int_{-\infty}^{\infty} ds\ \varphi(s)\ j^\alpha(x,\tau - s) \\ &= \int_{-\infty}^{\infty} ds\ \mathbf{P}_\lambda(s)\ \dot{X}^\alpha(\tau - s)\ \delta^4(x^\mu - X^\mu(\tau - s)) \end{aligned}$$

Current at τ produced by events delayed by randomly distributed time s

Laplace Distribution

$$\mathbf{P}_\lambda(s) = \frac{1}{2\lambda} e^{-|s|/\lambda}$$

Delay time distribution for Poisson processes