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Green's Functions for Off-Shell Electromagnetism and Spacelike Correlations

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Abstract. The requirement of gauge invariance for the Schwinger- DeWitt equations, interpreted as a manifestly covariant quantum theory for the evolution of a system in spacetime, implies the existence of a five dimensional pre-Maxwell field on the manifold of spacetime and "proper time" τ. The Maxwell theory is contained in this theory; integration of the field equations over τ restores the Maxwell equations with the usual interpretation of the sources. Following Schwinger's techniques, we study the Green's functions for the five dimensional hyperbolic field equations for both signatures $\pm$ (corresponding to O(4,1) or O(3,2) symmetry of the field equations) of the proper time derivative. The classification of the Green's functions follows that of the four dimensional theory for "massive" fields, for which the "mass" squared may be positive or negative, respectively. The Green's functions for the five dimensional field is then given by the Fourier transform over the "mass" parameter. We derive the Green's functions corresponding to the principal part Δ_P and the homogeneous function Δ_1; all of the Green's functions can be expressed in terms of these, as for the usual field equations with definite mass. In the O(3,2) case, the principal part function has support for $x^2 \geq \tau^2$, corresponding to spacelike propagation, as well as along the light cone $x^2 = 0$ (for $\tau = 0$). There can be no transmission of *information* in spacelike directions, with this propagator, since the Maxwell field, obtained by integration over τ, does not contain this component of the support. Measurements are characterized by such an integration. The spacelike field therefore can dynamically establish spacelike correlations.

1.Introduction

The method of Schwinger[1] and DeWitt[2] for obtaining the propagator for a quantum field consists of defining a Hilbert space $L^2(R^4)$ for which the canonical four momenta and coordinates, p^μ and x^μ, are represented by operators satisfying the commutation relations ($g^{\mu\nu} = diag(-,+,+,+)$)

$$[x^\mu, p^\nu] = ig^{\mu\nu}. \tag{1}$$

For a field equation of the form

$$K(-i\partial_\mu, x^\mu)\psi(x) = 0, \tag{2}$$

we may write the Green's function as (where we denote x^μ by x)

$$G(x,x') = i\int_0^\infty d\tau < x|e^{-iK(p,x)\tau}|x' > . \tag{3}$$

Multiplying the function $G(x,x')$ by the differential operator appearing in Eq. (2), one can bring it into the matrix element as the operator $K(p,x)$. This is equivalent to the action of the τ derivative of the integrand; the integral can be carried out by adding $-i\epsilon$ to K in the exponent (the existence of the integral in the formal expression (3) requires such a factor, or a suitable choice of test function for the integrand, viewed as a distribution) to obtain $\delta^4(x-x')$, i.e., the formula (3) is a valid representation of the Green's function.

The quantity

$$\psi_\tau(x) = \int d^4x' < x|e^{-iK(p,x)\tau}|x' > \phi(x') \tag{4}$$

satisfies the equation

$$\frac{i\partial\psi_\tau(x)}{\partial\tau} = K(p,x)\psi_\tau(x) \tag{5}$$

assumed by Stueckelberg[3] to describe the evolution of a wave function in the invariant parameter τ in a manifestly covariant quantum theory. The evolution equation for the function whose integral is the Green's function for the quantum field equations was obtained by Schwinger and DeWitt as a formal technical procedure. Stueckelberg was motivated to introduce the parametric description of the evolution of the classical world lines, and the quantum mechanical evolution (generating approximate world lines through the Ehrenfest motion of the wave packet), by the observation that pair annihilation can be described by a world line which reaches some region in finite spacetime from $t = -\infty$, and returns to $t = -\infty$ moving in the reverse direction of time. He remarked that the variable t is clearly not adequate to parametrize the motion in such a configuration, and therefore introduced the invariant parameter τ. Horwitz and Piron[4] extended the ideas of Stueckelberg (which can be traced back to Fock[5]) to the formulation of a theory applicable to many particles with electromagnetic interaction and action at a distance (in spacetime) potentials. It has been shown that the Newton-Wigner position operator is naturally contained in the structure of the theory, and the uncertainty relation of Landau and Peierls (interpreted by Aharonov and Albert as a causal requirement) follows from an operator commutation relation[5]. For the two body problem (with Lorentz invariant potential functions) the Schrödinger spectrum for nonrelativistic hydrogen is obtained for the *mass* spectrum of the relativistic problem; it reduces to the total mass plus the Schrödinger energy spectrum in the nonrelativistic limit[6]. The precise two body wave functions were found, and for the case of continuous (relative) spectrum, a relativistic partial wave expansion was found[7]. To study the radiative transitions of an atom, it is necessary to investigate the structure of the electromagnetic interaction.

The Maxwell field, which is independent of τ, has for its source the integral over τ of the current associated with Eq. (5), taking for the generator of motion a form quadratic in the energy-momentum vector (i.e., the gauge invariant form of $K = \frac{p_\mu p^\mu}{2M}$ [8]). This current satisfies the condition

$$\partial_\mu j^\mu + \frac{\partial \rho}{\partial \tau} = 0, \tag{6}$$

where $\rho = |\psi|^2$. Integrating over τ, the second term vanishes under appropriate boundary conditions [3,9]; hence the integral of the τ-dependent current j_μ , on $-\infty$ to ∞, has a vanishing four-divergence. The dynamical problem of the evolution of events is then, however, not well-conditioned. The motion of each of the events would depend on the entire world line of the other, which is responsible for the field in which it moves. The solution of the problem then requires an iteration to find a self-consistent field, a procedure which may not be stable[10].

A well-conditioned procedure can be formulated by introducing τ-dependent fields, with the five-dimensionally conserved τ-dependent current as its source[11]. Such a theory emerges naturally from the gauge invariant form of Eq. (5). The requirement of covariance in structure under the transformation

$$\psi' = e^{i\Lambda(x,\tau)}\psi \tag{7}$$

can be satisfied by introducing five gauge compensation fields, a_α, for $\alpha = 0, 1, 2, 3, 4$ (which we shall call pre-Maxwell fields), and write the equation as

$$(i\partial_\tau + e_0 a_4)\psi_\tau(x) = K(p - e_0 a, x)\psi_\tau(x), \tag{8}$$

where, under gauge transformation,

$$a_\alpha \to a_\alpha + \partial_\alpha \Lambda$$

preserves the form of the equation[11]. Green's functions for this type of equation have been extensively studied by Osborn and collaborators[12]. Since (as we shall show below) the Maxwell field is related to the potentials a_μ by integration over τ, a_μ has dimension $(length)^{-2}$. Hence, e_0 has the dimension of length. Second order field equations can be obtained (along with the equation of motion (5) for the wave function), by assuming a Lagrangian containing the term $(\frac{\lambda}{4}) f_{\alpha\beta} f^{\alpha\beta}$ (where λ also has the dimension of length, and $f_{\alpha\beta} \equiv \partial_\alpha a_\beta - \partial_\beta a_\alpha$), of the form[11]

$$\partial_\beta f^{\alpha\beta} = e j^\alpha, \tag{9}$$

where $e = \frac{e_0}{\lambda}$. Note that we have formally written the $\alpha = 4$ index as a subscript or superscript to conform with the covariant and contravariant forms of the Lorentz indices; we shall discuss the associated signature later.

This theory leads to a consistent, well-posed, dynamical problem, in which the particle world lines develop through the motion of the events under the influence of fields generated by the motion of other events in spacetime. Furthermore, integration of the equations (9) for the $\alpha = \mu$ components (with appropriate asymptotic conditions), reproduces the usual Maxwell equations with the τ-integrated currents as sources. Hence, *a posteriori*, the solution of the problem satisfies the requirements of the macroscopic Maxwell theory. The world lines, associated with the four dimensionally conserved currents, give rise to the Maxwell fields which emerge as solutions of the Maxwell wave equations with these sources. The difference between this theory and the Maxwell theory is the dynamical influence of the fields on the equations for the motion of the events; they follow the Lorentz forces associated with the five-dimensional pre-Maxwell fields which only approximate the Maxwell Lorentz forces for large τ (for continuous "mass"spectrum, the large τ limit restricts, by the Riemann-Lebesgue lemma, the contributions of the Fourier components of the field to those of small "mass", close to the Maxwell limit).

Returning to the Schwinger-DeWitt approach to the derivation of the underlying covariant quantum mechanics, we see that the requirement of gauge invariance for the Stueckelberg equation has led to τ-dependent fields, and, moreover, an additional field a_4. Their procedure can be recovered by defining operators $\mathbf{a}_\alpha$ which, acting on a direct sum space with elements

$$\psi(x) = i \int_0^\infty \psi_\tau(x) d\tau, \tag{10}$$

take on the values $a_\alpha(x, \tau)$ on each of the constituent τ-spaces. The corresponding field equation then becomes

$$(K(-i\partial_\mu - e_0\mathbf{a}_\mu, x) - e_0\mathbf{a}_4)\psi = 0; \tag{11}$$

the $\mathbf{a}_4$ term appears in the form of an explicit mass renormalization.

We now turn to a study of the Green's functions for the fields $f_{\alpha\beta}$. In Section 3, we discuss the causal properties of these functions, and show, in fact, that there is a class of solutions with relative spacelike support. These contributions vanish upon integration over τ. If we assume that detection apparatus necessarily involves integration over τ, so that only Maxwell type signals are directly observable, these contributions can only give rise to spacelike correlations, manifested by actual interaction, but not to the transfer of information.

It is a pleasure to dedicate this article to J.S. Bell, who has contributed much to our thinking about classical and quantum correlations.

2. Green's functions

The antisymmetric derivative of the field equations (9) results in[11]

$$(\partial_\mu\partial^\mu + \partial_\tau\partial^\tau) f^{\alpha\beta} = -e(\partial^\alpha j^\beta - \partial^\beta j^\alpha). \tag{12}$$

The operator on the right hand side also appears directly in Eq. (9) in the five dimensional Lorentz gauge $\partial_\alpha a^\alpha = 0$. Eq. (9), in this gauge becomes

$$\partial_\beta \partial^\beta a^\alpha = -e j^\alpha. \tag{13}$$

The spacetime Fourier transform of a source-free equation of the type (12) or (13), say, $\partial_\beta \partial^\beta f = 0$, is

$$(\partial_\tau \partial^\tau - k_\mu k^\mu) f(k,\tau) = 0; \tag{14}$$

if k^μ is spacelike (timelike), in order that the solutions $f(k,\tau)$ to be bounded for all τ, the second order τ-derivative should occur with a minus (plus) sign. The O(3,2) and O(4,1) choices of the signature of the τ-derivative therefore correspond to stable solutions of the source free field equations for spacelike and timelike four momenta of the off-shell fields respectively. To study the associated Green's functions, we shall write the operator generically as $\partial_\mu \partial^\mu + \sigma {\partial_\tau}^2$, where $\sigma = \pm$. As we shall see, the causal structure of the Green's functions is quite different in the two cases.

The Fourier transform of Eq.(14) over τ alone, with complementary variable κ, leads to a differential equation of the form of the Klein Gordon equation for a massive vector meson with "mass squared" $\sigma\kappa^2$. We shall construct and classify the Green's functions according the procedures used for the usual massive Klein-Gordon theory for each value of κ; the resulting functions $\Delta(x, \sigma\kappa^2)$, are analytic in κ^2, and hence this construction can be used for either value of σ. The final result is obtained by inverting the transform. The integral over τ, corresponding to $\kappa = 0$, reduces the Green's functions to the corresponding Maxwell propagators.

Let us define the Green's functions by the integral

$$\begin{aligned} G(x,\tau) &= \frac{1}{2\pi} \int d\kappa e^{i\sigma\kappa\tau} \Delta(x, \sigma\kappa^2) \\ &= \frac{1}{(2\pi)^5} \int_C d^4k d\kappa e^{i(k\cdot x + \sigma\kappa\tau)} \frac{1}{k^2 + \sigma\kappa^2}, \end{aligned} \tag{15}$$

where the appropriate type and boundary conditions are determined by the choice of contour C for the integration of k^0. Recall that the functions $\Delta_A(x, \sigma\kappa^2)$ and $\Delta_R(x, \sigma\kappa^2)$ (we use the notation and conventions of Jauch and Rohrlich[13]) are the usual advanced and retarded Green's functions with support, respectively, in the forward and backward light cones (for σ positive). For $\kappa^2 = 0$, they have support on the light cone and are used in obtaining the Lienard Wiechert potentials in classical radiation theory. The function $\Delta_{1R}(x, \sigma\kappa^2)$ is Feynman's Green's function, and has no classical analog. Finally, $\Delta_P(x, \sigma\kappa^2)$ is associated with the principal part. All of the Green's functions can be related to Δ_P and Δ_1 with the factors $\theta(x)$ and $\varepsilon(x)$, and linear combinations. Schwinger[14] has given very useful explicit representations for these Green's functions; with only slight modifications because of the sign of σ, our derivation follows his until the final integrations on κ.

We begin with the principal part function

$$\Delta_P(x, \sigma\kappa^2) = -\frac{i}{2} (\frac{1}{2\pi})^4 \int da \varepsilon(a) \int d^4k \exp i(ak^2 + k\cdot x + a\sigma\kappa^2), \tag{16}$$

where we have used the identity

$$P(\frac{1}{k^2+\sigma\kappa^2}) = -\frac{i}{2}\int_{-\infty}^{+\infty} da\varepsilon(a)e^{ia(k^2+\sigma\kappa^2)}. \tag{17}$$

Using the result

$$\int d^4k e^{i(ak^2+k\cdot x)} = (i\frac{\pi^2}{a^2})\varepsilon(a)e^{-i(\frac{x^2}{4a})},$$

we obtain

$$\Delta(x,\sigma\kappa^2) = \frac{1}{8(2\pi)^2}\int \frac{da}{a^2}exp(-\frac{ix^2}{4a}+ia\sigma\kappa^2). \tag{18}$$

We now introduce the change of variables $\alpha = 1/4a$. It then follows (with a little care at $a=0$)that

$$\Delta_P(x,\sigma\kappa^2) = \frac{1}{2(2\pi)^2}\int_{-\infty}^{+\infty} d\alpha e^{i(-x^2\alpha+\frac{\sigma\kappa^2}{4\alpha})}. \tag{19}$$

Since the integration on $d\alpha$ is over symmetric limits, only the cosine term remains. Writing $\lambda = -x^2$, we have

$$\begin{aligned}\Delta_P(x,\sigma\kappa^2) &= \frac{1}{(2\pi)^2}\int_0^{\infty} d\alpha\cos(\lambda\alpha+\sigma\kappa^2/4\alpha) \\ &= \frac{1}{(2\pi)^2}\frac{\partial}{\partial\lambda}\int_0^{\infty}\frac{d\alpha}{\alpha}\sin(\lambda\alpha+\sigma\kappa^2/4\alpha).\end{aligned} \tag{20}$$

We now replace α by $(\sigma\kappa/2|\lambda|^{1/2})e^{\theta}$ to obtain

$$\Delta_P(x,\sigma\kappa^2) = \int_{-\infty}^{+\infty} d\theta\sin[\frac{\kappa|\lambda|^{1/2}}{2}(\varepsilon(\lambda\sigma)e^{\theta}+e^{-\theta})]. \tag{21}$$

Thus, we have that

$$\Delta_P(x,\sigma\kappa^2) = \begin{cases}\int_{-\infty}^{\infty} d\theta\sin[\kappa|\lambda|^{1/2}\cosh\theta], & \lambda\sigma>0 \\ -\int_{-\infty}^{\infty} d\theta\sin[\kappa|\lambda|^{1/2}\sinh\theta], & \lambda\sigma<0,\end{cases} \tag{22}$$

or,

$$\Delta_P(x,\sigma\kappa^2) = \frac{1}{(2\pi)^2}\frac{\partial}{\partial\lambda}\begin{cases}\pi J_0(\kappa|\lambda|^{1/2}), & \lambda\sigma>0 \\ 0, & \lambda\sigma<0.\end{cases} \tag{23}$$

Restoring $\lambda = -x^2$, we find that (we use a notation which identifies the spacetime and τ-dependent Green's functions with the corresponding Δ's)

$$G_P(x,\tau) = -\frac{1}{8\pi^2}\int d\kappa e^{i\sigma\kappa\tau}[\delta(x^2) + \theta(-\sigma x^2)\frac{\partial}{\partial x^2}J_0(\kappa|x^2|^{1/2})]. \tag{24}$$

Carrying out the Fourier transform, we obtain

$$G_P(x,\tau) = -\frac{1}{4\pi}\delta(x^2)\delta(\tau) - \frac{1}{(2\pi)^2}\frac{\partial}{\partial x^2}\begin{cases}\frac{\theta(x^2-\tau^2)}{\sqrt{x^2-\tau^2}}, & \sigma = -1;\ \mathrm{O(3,2)}\\ \frac{\theta(-x^2-\tau^2)}{\sqrt{-x^2-\tau^2}}, & \sigma = +1;\ \mathrm{O(4,1)}.\end{cases} \tag{25}$$

This may be written more compactly as

$$G_P(x,\tau) = -\frac{1}{4\pi}\delta(x^2)\delta(\tau) - \frac{1}{2\pi^2}\frac{\partial}{\partial x^2}\theta(-\sigma g_{\alpha\beta}x^\alpha x^\beta)\frac{1}{\sqrt{-\sigma g_{\alpha\beta}x^\alpha x^\beta}}. \tag{26}$$

To obtain an explicit expression for $G_1(x,\tau)$, we define

$$\Delta_1 = \frac{1}{(2\pi)^3}\int d^4k e^{ik\cdot x}\delta(k^2 + \sigma\kappa^2), \tag{27}$$

which coincides with the usual expression for $\sigma = +$. Following a method similar to the one used above (i.e., starting with Eq. (16) without the factor $\varepsilon(a)$), we find that

$$\Delta_1(x,\sigma\kappa^2) = \frac{1}{2\pi^2}\frac{\partial}{\partial\lambda}\int_0^\infty \frac{d\alpha}{\alpha}\cos(\lambda\alpha + \frac{\sigma\kappa^2}{4\alpha}), \tag{28}$$

where we note that the sine in (20) has been replaced by the cosine. Making the same replacement for α as in going from (20) to (21), we obtain

$$\Delta_1(x,\sigma\kappa^2) = \int_{-\infty}^{+\infty} d\theta \cos\,[(\kappa|\lambda|^{1/2})\begin{Bmatrix}\cosh\theta\\ \sinh\theta\end{Bmatrix}], \tag{29}$$

for, respectively, $\sigma\lambda$ greater or less than zero. The result is

$$\Delta_1(x,\sigma\kappa^2) = \frac{1}{2\pi^2}\frac{\partial}{\partial\lambda}\begin{cases}-\pi N_0(\kappa|\lambda|^{1/2}), & \lambda\sigma > 0\\ 2K_0(\kappa|\lambda|^{1/2}), & \lambda\sigma < 0.\end{cases} \tag{30}$$

Note that there is no singularity as $|\lambda| \to 0$. Carrying out the remaining Fourier tranform over κ as in Eq. (24), we obtain

$$G_1(x,\tau) = \frac{1}{2\pi^2}\frac{\partial}{\partial x^2}\begin{cases}\frac{\theta(\tau^2-|x|^2)}{\sqrt{\tau^2-|x|^2}}, & \sigma x^2 < 0\\ \frac{1}{\sqrt{\tau^2+|x|^2}} & \sigma x^2 > 0.\end{cases} \tag{31}$$

For the O(3,2) case ($\sigma = -$) we have

$$G_1(x,\tau) = \frac{1}{2\pi^2}\frac{\partial}{\partial x^2}\frac{\theta(\tau^2 - x^2)}{\sqrt{\tau^2 - x^2}}, \tag{32}$$

while for O(4,1) we have

$$G_1(x,\tau) = \frac{1}{2\pi^2}\frac{\partial}{\partial x^2}\frac{\theta(\tau^2 + x^2)}{\sqrt{\tau^2 + x^2}}. \tag{33}$$

These expressions may be combined as

$$G_1(x,\tau) = \frac{1}{2\pi^2}\frac{\partial}{\partial x^2}\frac{\theta(\sigma g_{\alpha\beta}x^\alpha x^\beta)}{\sqrt{\sigma g_{\alpha\beta}x^\alpha x^\beta}}. \tag{34}$$

3. Causal properties

The propagators $\Delta_R(x,m^2)$, $\Delta_A(x,m^2)$ and $\Delta(x,m^2)$, which may be obtained from Δ_P by multiplication by $\theta(\pm x)$ [13], for massive fields, vanish outside of the light cone, and in the massless photon case have support only on the light cone. The massless limit of the first two are used to obtain the Lienard-Wiechart potentials, and the third is associated with the propagation of radiation.

The Green's function $G_P(x,\tau)$ (Eq.(26)) contains two terms. The first contributes only at $\tau = 0$(equal τ between the two events connected by the Green's function); the second for the entire range $|x^2| > \tau^2$, corresponding to spacelike intervals for the O(3,2) case and timelike intervals for the O(4,1) case. Integrating over the τ variable associated with the range of the kernel (i.e.,τ_1 associated with x_1), as would occur as an argument in the field associated with a source by means of the Green's function, reduces the contribution of the first term to a Maxwell type propagator acting on a τ-integrated current, i.e.,

$$\begin{aligned}\int d\tau_1 a_\mu(x_1,\tau_1) &= \int d\tau_1 d^4x_2 \delta((x_1 - x_2)^2)\delta(\tau_1 - \tau_2) j_\mu(x_2,\tau_2)\\ &= \int d^4x_2 \delta((x_1 - x_2)^2) J_\mu(x),\end{aligned}$$

where $J_\mu(x)$ is the four-dimensionally conserved current. The contribution of the second term vanishes under integration over τ_1 (the integral over τ is independent of x^2). Dynamical interactions, however, take place through the τ-dependent fields. The second term contributes to the dynamical evolution in τ_1 of a charged event at the point x_1. In the O(3,2) case this interaction corresponds to an effect occurring outside of the causal light cone. We argue that this effect does not violate the interdiction on the transmission of information at speeds greater than light velocity. Detection apparatus is designed to respond to signals of Maxwell type; from the point of view of the theory that we have formulated here, the detection apparatus must integrate over τ. *We shall assume that our perception of radiation, and the detection of electromagnetic signals by measurement apparatus, necessarily involves integration over* τ. Hence, the acausal components of the Green's function can be effective in dynamical interaction, but they cannot carry information. Such effects would appear phenomenologically as *correlations.*

The O(3,2) principal part propagator has the property that two points in spacetime, with some given spacelike interval, are not connected for $|\tau_1 - \tau_2|$ larger than that interval. This corresponds to an approximately equal τ restriction on interaction, for which an action at a distance potential is an idealization.

In the O(4,1) case, the functions $G_P(x,\tau)$ have support in the timelike region for $-x^2 > \tau^2$; the wave equation, in Fourier transform, has a positive "mass-squared" term. Contributions of this type correspond, therefore, to massive vector fields.

The $G_1(x,\tau)$ functions, which occur in the quantum evolution problem (the Feynman propagator is defined by $\Delta_{1R} = \Delta_P + \frac{i}{2}\Delta_1$) has support for $|x^2| < \tau^2$ in the spacelike region for the O(3,2) case, where it has singular behavior at the end of its range, and in the full timelike region, where it is regular and falls off asymptotically as $1/\rho^3$. One may consider the bounded range in the spacelike direction as a "velocity" (of the form $d\rho/d\tau$) limited evolution. In the O(4,1) case, the structure of the support is the same, but with interchange of the timelike and spacelike behavior.

Conclusions

We have studied the propagators for the five dimensional wave equations arising from the requirement of gauge invariance of the O(3,1) invariant Stueckelberg-Schwinger-DeWitt equation. These equations have a formal O(3,2) or O(4,1) invariance, depending on the choice of signature for $d\tau$.

We find that the O(3,2) principal part Green's function, associated with radiation and the generalized Lorentz force(we shall discuss the latter in a succeeding paper), has spacelike ($x^2 > \tau^2$) support which can lead, through interaction, to correlation but not to faster than light propagation of information. Action at a distance potentials (at equal τ) provide an approximate idealization of this type of propagation. To reach this conclusion, we assumed that detection apparatus always integrates over τ, accounting for our perception of the Maxwell fields. One arrives at a picture of the evolution of a two body system, for example, in which each responds to the field of the other by a generalized (pre-Maxwell) Lorentz force (or, quantum mechanically, the minimal gauge interaction). *A posteriori*, the integrated pre-Maxwell field that results agrees precisely with the Maxwell field associated with the world lines, but the motion does not exactly follow the Maxwell-Lorentz force microscopically. For large τ differences, the field asymptotically (by Riemann-Lesbesgue) contains only values of κ close to zero, and hence the macroscopic Lorentz forces are in close agreement with those of the Maxwell theory. One might expect that the structure of the radiation reaction problem for a single charged particle, involving microscopic distances, and high frequency components of Compton scattering, will be somewhat different from that predicted by the Maxwell theory. These phenomena are currently under investigation.

In these arguments, we have implicitly assumed the existence of some kind of scale of what is large and small. We have introduced dimensional couplings for the off-shell fields; the purely electromagnetic equations depend only on their ratio, corresponding to the dimensionless Maxwell charge. The energy momentum tensor for the field and the pre-Maxwell Lorentz forces, however, contain these dimensional constants. These scales, and the associated mass shell deviations of the fields during interaction, necessarily determine the notions of large and small.

The $G_1(x,\tau)$ function for O(3,2) (associated to quantum phenomena through the Feynman propagator $G_P + \frac{i}{2}G_1$) has, on the other hand, support for $x^2 < \tau^2$ (including negative x^2 as well). One may think of this bound as a generalized type of "causal" structure, in which the spacelike distance over which the propagator acts cannot exceed the invariant τ. Indeed, an alternative construction of the Green's function, obtained by carrying out the integration over κ first, results in a representation in terms of the integral over a hyperbolic variable (a rapidity parameter) of the delta functions $\delta(\sqrt{|x^2|}cosh\beta \pm \tau)$, for which each contribution corresponds to a generalized "causal" wavefront.

Similar statements can be made with respect to the O(4,1) Green's functions, with the role of space and time interchanged. The principal part Green's function has no support in the spacelike sector. The timelike contributions vanish upon integration over τ; the remaining Green's function, with lightlike support, coincides with the residual support of the O(3,2) function and the Maxwell propagator. The G_1 function has bounded support for $-x^2 < \tau^2$ in the timelike sectors, but only falls off as $1/\rho^3$ in the spacelike sector. Upon integration over τ, the Green's functions $G_P + \frac{i}{2}G_1$ reduce to the standard Feynman propagator for the massless electromagnetic field in both the O(3,2) and O(4,1) cases. With the assumption that detection apparatus integrates over τ, this assures that these propagators cannot transfer *information* faster than the speed of light; the dynamical interactions they mediate are necessarily seen only as *correlations*.

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