

Higher-Order Kinetic Term for Controlling Photon Mass in Off-Shell Electrodynamics

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Abstract

In relativistic classical and quantum mechanics with Poincare-invariant parameter, particle worldlines are traced out by the evolution of spacetime events. The formulation of a covariant canonical framework for the evolving events leads to a dynamical theory in which mass conservation is demoted from a *a priori* constraint to the status of conserved Noether current for a certain class of interactions. In pre-Maxwell electrodynamics — the local gauge theory associated with this framework — events induce five local off-shell fields, which mediate interactions between instantaneous events, not between the worldlines which represent entire particle histories. The fifth field, required to compensate for dependence of gauge transformations on the evolution parameter, enables the exchange of mass between particles and fields. In the equilibrium limit, these pre-Maxwell fields are pushed onto the zero-mass shell, but during interactions there is no mechanism regulating the mass that photons may acquire, even when event trajectories evolve far into the spacelike region. This feature of the off-shell formalism requires the application of some *ad hoc* mechanism for controlling the photon mass in two opposite physical domains: the low energy motion of a charged event in classical Coulomb scattering, and the renormalization of off-shell quantum electrodynamics. In this paper, we discuss a nonlocal, higher derivative correction to the photon kinetic term, which provides regulation of the photon mass in a manner which preserves the gauge invariance and Poincare covariance of the original theory. We demonstrate that the inclusion of this term is equivalent to an earlier solution to the classical Coulomb problem, and that the resulting quantum field theory is renormalized.

1 Introduction

1.1 Pre-Maxwell Theory

The Stueckelberg equation [1, 2]

$$i\partial_\tau\psi(x, \tau) = \frac{1}{2M}p^\mu p_\mu\psi(x, \tau) , \quad (1)$$

describing a free particle in the quantum mechanics of Horwitz and Piron [2], is rendered locally gauge invariant [3] under gauge transformations

$$\psi(x, \tau) \longrightarrow [\exp ie_0\Lambda(x, \tau)] \psi(x, \tau) \quad (2)$$

when extended to the form

$$(i\partial_\tau + e_0a_5) \psi(x, \tau) = \frac{1}{2M}(p^\mu - e_0a^\mu)(p_\mu - e_0a_\mu) \psi(x, \tau) , \quad (3)$$

with gauge compensation fields which transform as

$$a_\mu(x, \tau) \rightarrow a_\mu(x, \tau) + \partial_\mu\Lambda(x, \tau) \quad a_5(x, \tau) \rightarrow a_5(x, \tau) + \partial_\tau\Lambda(x, \tau) . \quad (4)$$

Adopting the conventions

$$x^5 = \tau \quad \partial_5 = \partial_\tau \quad \lambda, \mu, \nu = 0, 1, 2, 3 \quad \alpha, \beta, \gamma = 0, 1, 2, 3, 5 \quad (5)$$

reframes equation (4) as a five-dimensional symmetry transformation

$$a_\alpha(x, \tau) \rightarrow a_\alpha(x, \tau) + \partial_\alpha\Lambda(x, \tau) \quad (6)$$

inducing — from equation (3) — a five-dimensional conserved current

$$\partial_\alpha j^\alpha = 0 \quad (7)$$

where

$$j^5 = |\psi(x, \tau)|^2 \quad j^\mu = \frac{-i}{2M} \{ \psi^* (\partial^\mu - ie_0a^\mu) \psi - \psi (\partial^\mu + ie_0a^\mu) \psi^* \} . \quad (8)$$

Equations (7) and (8) permit the interpretation of $|\psi(x, \tau)|^2$ as the probability density at τ of finding the event at x . Since $\partial_\mu j^\mu = -\partial_\tau j^5 \neq 0$, we may not identify j^μ as the source current in Maxwell's equations. However, under the boundary conditions $j^5 \rightarrow 0$, pointwise, as $\tau \rightarrow \pm\infty$, integration of (7) over τ , leads to $\partial_\mu J^\mu = 0$, where

$$J^\mu(x) = \int_{-\infty}^{\infty} d\tau j^\mu(x, \tau) . \quad (9)$$

This integration has been called concatenation [4] and links the event current j^μ with the particle current J^μ defined on the entire worldline. The quantum mechanical potential

theory with $a_\mu = 0$ and $-e_0\phi = V(\sqrt{x^\mu x_\mu})$ has been solved for the standard bound state [5] and scattering [6] problems. Some justification is provided for the pre-Maxwell theory by the covariant treatment of the Zeeman [7] and Stark [8] effects for the hydrogen-like bound state: the Horwitz-Piron theory reproduces the expected line splitting phenomenology, only when the pre-Maxwell scalar current is included.

The associated classical mechanics is obtained by transforming the Hamiltonian found from (3) to a classical Lagrangian [9], and including the gauge invariant kinetic term for the fields proposed by Sa'ad, et. al. [3]:

$$L = \frac{1}{2}M\dot{x}^\mu\dot{x}_\mu + e_0\dot{x}^\alpha a_\alpha - \frac{\lambda}{4}f^{\alpha\beta}f_{\alpha\beta} . \quad (10)$$

The field strength tensor

$$f_{\alpha\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha \quad (11)$$

is chosen — as in standard Maxwell theory — to be first order in the fields and manifestly gauge invariant. Variation of (10) with respect to x^μ leads to the classical Lorentz force [9]

$$M\ddot{x}^\mu = e_0 f^\mu{}_\alpha(x, \tau)\dot{x}^\alpha \quad \frac{d}{d\tau}\left(-\frac{1}{2}M\dot{x}^2\right) = e_0 f_{5\alpha}\dot{x}^\alpha . \quad (12)$$

and exchange of mass between particles and fields may be seen in the second of equation (12); the total mass-energy-momentum of the events and fields is, however, conserved [9]. Since particle mass is not separately conserved, pair annihilation is classically permitted.

In formally raising the index $\beta = 5$ in $f^{\mu 5} = \partial^\mu a^5 - \partial^5 a^\mu$, Sa'ad et. al. argue that the action suggests a higher symmetry containing $O(3,1)$ as a subgroup, that is, either $O(4,1)$ or $O(3,2)$. They wrote the metric for the field as

$$g^{\alpha\beta} = \text{diag}(-1, 1, 1, 1, \sigma) , \quad (13)$$

where $\sigma = \pm 1$, depending on the higher symmetry. Variation of (10) with respect to a_α yields

$$\partial_\beta f^{\alpha\beta} = \frac{e_0}{\lambda}j^\alpha = e j^\alpha \quad \epsilon^{\alpha\beta\gamma\delta\epsilon}\partial_\alpha f_{\beta\gamma} = 0 \quad (14)$$

where e_0/λ is identified as the dimensionless charge e , and the current j^α associated with an event $X^\alpha = (X^\mu(\tau), \tau)$ is given by

$$j^\alpha(x, \tau) = \frac{dX^\alpha}{d\tau}\delta^4(x^\mu - X^\mu(\tau)) . \quad (15)$$

At the quantum level, the current is given by (8).

In analogy to the concatenation of the current in (9), we see that under the boundary conditions $f^{5\mu} \rightarrow 0$, pointwise as $\tau \rightarrow \pm\infty$, we recover the Maxwell fields from the pre-Maxwell fields as

$$\partial_\nu F^{\mu\nu} = eJ^\mu \quad \epsilon^{\mu\nu\rho\lambda} \partial_\mu F_{\nu\rho} = 0 \quad (16)$$

where

$$F^{\mu\nu}(x) = \int_{-\infty}^{\infty} d\tau f^{\mu\nu}(x, \tau) \quad \text{and} \quad A^\mu(x) = \int_{-\infty}^{\infty} d\tau a^\mu(x, \tau) . \quad (17)$$

Since $e_0 a_\mu$ and eA_μ must have the same dimensions, it follows from (17) that λ (and hence $e_0 = \lambda e$) must have dimensions of time. Although the parameter λ does not appear in the field equations (14), it does appear in the Lorentz force (12) through e_0 . The presence of this dimensional parameter in the equations of motion is a characteristic problem in the classical formalism.

Although the physical Lorentz covariance of the current j^α breaks the higher symmetry of the free field equations to $O(3,1)$, the wave equation

$$\partial_\alpha \partial^\alpha a^\beta = (\partial_\mu \partial^\mu + \partial_\tau \partial^\tau) a^\beta = (\partial_\mu \partial^\mu + \sigma \partial_\tau^2) a^\beta = -\frac{e}{c} j^\beta , \quad (18)$$

reflects the causal properties of the higher symmetry through the operator on the left hand side. The classical Green's function for (18), defined through

$$\partial_\alpha \partial^\alpha G(x, x^5) = -\delta^4(x, x^5) , \quad (19)$$

is given by [10]

$$G(x, x^5) = -\frac{1}{4\pi} \delta(x^2) \delta(x^5) - \frac{1}{2\pi^2} \frac{\partial}{\partial x^2} \frac{\theta(-\sigma g_{\alpha\beta} x^\alpha x^\beta)}{\sqrt{-\sigma g_{\alpha\beta} x^\alpha x^\beta}} . \quad (20)$$

It follows from (18) and (19) that the potential induced by a known current is given by

$$a^\beta(x, \tau) = -e \int d^4x' d\tau' G(x - x', \tau - \tau') j^\beta(x', \tau') . \quad (21)$$

Under concatenation, the first term of (20) becomes the Maxwell Green's function

$$D(x) = -\frac{1}{4\pi} \delta(x^2) , \quad (22)$$

while the second term — which induces spacelike or timelike correlations [10], depending on the signature σ — vanishes. This concatenation guarantees that the Maxwell potential is related to the Maxwell current in the usual manner:

$$\begin{aligned}
A^\mu(x) &= \int d\tau a^\mu(x, \tau) \\
&= -e \int d\tau \int d^4x' d\tau' G(x - x', \tau - \tau') j^\mu(x', \tau') \\
&= -e \int d^4x' d\tau' \left[\int d\tau G(x - x', \tau - \tau') \right] j^\mu(x', \tau') \\
&= -e \int d^4x' D(x - x') J^\mu(x') .
\end{aligned} \tag{23}$$

Therefore, we will refer to the Maxwell and the correlation terms of the Green's function and the induced potentials.

The off-shell quantum electrodynamics, associated with the action

$$S = \int d^4x d\tau \left\{ \psi^* (i\partial_\tau + e_0 a_5) \psi - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0 a_\mu) (-i\partial^\mu - e_0 a^\mu) \psi - \frac{\lambda}{4} f_{\alpha\beta} f^{\alpha\beta} \right\} , \tag{24}$$

has been worked out [11]. Manifestly covariant quantization has been given canonically [12, 11] and in path integral form [13, 11], and the perturbation theory developed [11]. The Feynman rules have been used to calculate the scattering cross section for two identical particles; this cross section reduces to the standard Klein-Gordon expression when no mass exchange is permitted [11], but for non-zero mass exchange, the forward and reverse poles each split into two and move away from the 0 and 180 degree directions. The off-shell quantum electrodynamics is counter-term renormalizable when the photon mass spectrum is cut off; without the cut-off, the mass integration in the loops cannot be controlled. We will see below that this cut-off may be introduced through a the higher order kinetic term introduced below.

1.2 Classical Coulomb Problem

The classical Coulomb problem was studied in the framework of the classical off-shell electromagnetic theory, in [14]. Posing the classical equations of motion for an event moving in the field induced by a second 'static' event (an event moving uniformly along the time

axis) the straightforward solution was seen to present a number of difficulties. First, the events can only interact under very specific circumstances, leading to a general picture of co-moving, non-interacting charged particles. Second, under interaction, the particle motion was shown to be piecewise linear, rather than smooth. The mathematical difficulties derive from the form of the classical current, which can be seen from (15) to have delta-function support on the worldline of the event. Thus, the source event

$$X^0(\tau) = \tau \quad \vec{X}(\tau) = 0 \quad (25)$$

induces the current

$$j^0(x, \tau) = j^5(x, \tau) = \frac{dX^0}{d\tau} \delta(x^0 - \tau) \delta^3(\vec{x}) = \delta(t - \tau) \delta^3(\vec{x}) , \quad (26)$$

with $\vec{j}(x, \tau) = 0$. Moreover, the leading term in the Green's function (20) enforces absolute simultaneity in τ , and reproduces a delta-function in the potential

$$a^0(x, \tau) = a^5(x, \tau) = -\frac{e}{4\pi R} \frac{1}{2} [\delta(t - R - \tau) + \delta(t + R - \tau)] \quad (27)$$

$$\vec{a}(x, \tau) = 0 \quad (28)$$

where $R = |\vec{x}|$. The second term in the Green's function makes no contribution for non-accelerated sources [14]. As required for the Maxwell part,

$$A^0(x) = \int d\tau a^0(x, \tau) = -\frac{e}{4\pi R} , \quad (29)$$

the concatenated potential has the form of the Coulomb potential induced by a ‘‘fixed’’ source. Nevertheless, according to the underlying pre-Maxwell dynamics, as given in the classical case by the Lorentz force equations

$$M \ddot{x}^0 = e_0 f^{0\alpha} \dot{x}_\alpha = -\lambda e [(\partial_k a^0) \dot{x}^k + \sigma(\partial_0 + \sigma \partial_\tau) a^0] \quad (30)$$

$$M \ddot{x}^k = e_0 f^{k\alpha} \dot{x}_\alpha = -\lambda e (\partial_k a^0) (\dot{x}^0 - \sigma) , \quad (31)$$

a test event moving in the field of this ‘static’ event will experience no interaction unless it crosses through $t \pm R - \tau = 0$, and in most cases the source and test event will appear co-moving and non-interacting in concatenated laboratory observations.

Naturally, the interaction condition depends upon the origin of motion for the source, and a different condition would apply if the source had been parameterized as $X^0(\tau) = \tau - \varepsilon$. In the classical context, by choosing a parameterization for the source and test events, $x_1^\mu(\tau)$ and $x_2^\mu(\tau)$ using a common τ , we implicitly impose a τ -correlation on the interaction through the τ -simultaneity of the Green's function. These difficulties do not arise in off-shell quantum mechanics, because the assumption of sharp asymptotic mass states leads to complete uncertainty in regard to the location of any event with respect to τ , and hence to the τ -correlation of a pair of events.

In order to arrive at a reasonable solution to the classical Coulomb problem, a distribution function was applied *ad hoc* to the field-inducing current in [14], in order to smooth out the behavior of the induced field. It was shown that the smoothed current

$$j_\varphi^\alpha(x, \tau) = \int_{-\infty}^{\infty} ds \varphi(\tau - s) j^\alpha(x, s) \quad (32)$$

in which φ is the Laplace distribution

$$\varphi(\tau) = \frac{1}{2\lambda} e^{-|\tau|/\lambda} \quad (33)$$

leads to a classical Yukawa potential, with interpretation of λ as the mass spectrum of the photons mediating the interaction. Moreover, the extra factor of $1/\lambda$ combines with e_0 in the Lorentz force equations (12) to provide the correct dimensionless coupling e . Since the smoothing distribution satisfies

$$\int_{-\infty}^{\infty} d\tau \varphi(\tau) = 1 \quad (34)$$

the concatenated Maxwell current is not affected by the integration (32).

The current associated with a 'static' particle is now spread out along the time axis as

$$j_\varphi^0(x, \tau) = \int_{-\infty}^{\infty} d\alpha \varphi(\alpha) j^0(x, \tau - \alpha) = \delta^3(\vec{x}) \varphi(t - \tau) , \quad (35)$$

as is the pre-Maxwell potential induced by this current

$$a_\varphi^0(x, \tau) = -\frac{e}{4\pi R} \frac{1}{2} [\varphi(t - R - \tau) + \varphi(t + R - \tau)] . \quad (36)$$

Under the Laplace distribution, (33) becomes

$$a_\varphi^0(x, \tau) = -\frac{e}{4\pi R} \frac{1}{2\lambda} \frac{1}{2} \left[e^{-|t-R-\tau|/\lambda} + e^{-|t+R-\tau|/\lambda} \right] , \quad (37)$$

and the motion of the test event under this potential coincides with classical Coulomb scattering for a small enough limit λ on the mass spectrum of the photons.

Solving the resulting equations of motion in this model, it was shown that trajectories are indistinguishable from the Maxwell case when $\lambda > 10^{-6}$ seconds, corresponding to a photon mass $m_\gamma = 1/\lambda \sim 10^{-9}$ eV. If we take the accepted experimental error in the photon mass as the actual mass of the photon, then $m_\gamma \simeq 6 \times 10^{-16}$ eV [15], which corresponds to $\lambda \simeq 1$ second.

Two closely related interpretations of the smoothing process in (35) were proposed in [14]. According to the first interpretation, the smoothing represents an alternative model for the relationship between events and particles, in which particle currents are described as a distribution of event currents with different initial conditions in τ . In the second interpretation, an uncertainty must be introduced in the mutual τ -correlation of the events. While these interpretations may hint at some potentially interesting notion of ‘classical decoherence’ and the Laplace distribution is suggestive of some Poisson process underlying the formation of particle currents, the mechanism remains *ad hoc* and leaves no clear path to quantization.

1.3 Nonlocal Electromagnetic Kinetic Term

In this paper, we return to classical pre-Maxwell electromagnetic theory, and propose a more general mechanism for smoothing the event current, involving a modification of the kinetic term for the electromagnetic field. We discuss the possible alternatives and choose a method which preserves Lorentz and gauge invariance of the action, and permits first and second quantization.

2 A Modified Electromagnetic Action

The classical electromagnetic action may be taken from (10) to be

$$S_{em} = \int d^4x d\tau \left[e_0 j^\alpha a_\alpha - \frac{\lambda}{4} f^{\alpha\beta} f_{\alpha\beta} \right] \quad , \quad (38)$$

and it would be convenient to make the replacement

$$j^\alpha(x, \tau) \rightarrow j_\varphi^\alpha(x, \tau) = \int_{-\infty}^{\infty} ds \varphi(\tau - s) j^\alpha(x, s) \quad (39)$$

in (38), leading directly to modified field equations

$$\partial_\beta f^{\alpha\beta}(x, \tau) = e j_\varphi^\alpha(x, \tau) \quad , \quad (40)$$

in which the fields are induced by the smoothed currents. Equation (40) reminds us to check for current conservation explicitly. Integrating by parts and taking the distribution to be even $\varphi(-\tau) = \varphi(\tau)$, we indeed find

$$\begin{aligned} \partial_\alpha j_\varphi^\alpha(x, \tau) &= \partial_\alpha \int_{-\infty}^{\infty} ds \varphi(\tau - s) j^\alpha(x, s) \\ &= \partial_\mu \int_{-\infty}^{\infty} ds \varphi(\tau - s) j^\mu(x, s) + \partial_\tau \int_{-\infty}^{\infty} ds \varphi(\tau - s) j^5(x, s) \\ &= \int_{-\infty}^{\infty} ds \varphi(\tau - s) \partial_\mu j^\mu(x, s) - \int_{-\infty}^{\infty} ds \partial_s \varphi(\tau - s) j^5(x, s) \\ &= \int_{-\infty}^{\infty} ds \varphi(\tau - s) \partial_\mu j^\mu(x, s) + \int_{-\infty}^{\infty} ds \varphi(\tau - s) \partial_s j^5(x, s) \\ &= \int_{-\infty}^{\infty} ds \varphi(\tau - s) [\partial_\mu j^\mu(x, s) + \partial_s j^5(x, s)] \\ \partial_\alpha j_\varphi^\alpha(x, \tau) &= 0 \quad . \end{aligned} \quad (41)$$

As usual, current conservation guarantees classical gauge invariance under (6) as

$$\begin{aligned} \int d^4x d\tau a_\alpha(x, \tau) j_\varphi^\alpha(x, \tau) &\rightarrow \int d^4x d\tau a_\alpha(x, \tau) j_\varphi^\alpha(x, \tau) \\ &\quad + \int d^4x d\tau \partial_\alpha \Lambda(x, \tau) j_\varphi^\alpha(x, \tau) \\ \int d^4x d\tau \partial_\alpha \Lambda(x, \tau) j_\varphi^\alpha(x, \tau) &= \int d^4x d\tau \partial_\alpha [\Lambda(x, \tau) j_\varphi^\alpha(x, \tau)] \\ &\quad - \int d^4x d\tau \partial_\alpha \Lambda(x, \tau) [\partial_\alpha j_\varphi^\alpha(x, \tau)] \\ &= 0 \end{aligned} \quad (42)$$

Although the replacement in (39) is manifestly O(3,1) covariant and gauge invariant, it is not clear how it may be applied in the quantum case (24), when the event current is formed from bilinear combinations of the field amplitudes. Moreover, the underlying logic of gauge theory requires that we regard the matter amplitudes $\psi(x, \tau)$ as more fundamental than the gauge fields $a_\alpha(x, \tau)$, and hence the form of the matter part of the action should be respected.

On the other hand, the kinetic term $\frac{\lambda}{4} f^{\alpha\beta} f_{\alpha\beta}$ for the gauge fields was introduced on essentially formal grounds — providing a term with first order derivatives of the fields, which is a Lorentz scalar, and gauge invariant. Because the field strengths $f^{\alpha\beta}$ are themselves gauge invariant, a kinetic term of the form

$$-\frac{\lambda}{4} \int d^4x d\tau ds f^{\alpha\beta}(x, \tau) \Phi(\tau - s) f_{\alpha\beta}(x, s) \quad (43)$$

retains the invariances of (38), with no assumptions on the symmetry properties of the distribution $\Phi(\tau)$ — the symmetric integration in (43) will vanish over the anti-symmetric part of the distribution function. The action

$$S_{em} = \int d^4x d\tau \left[e_0 j^\alpha(x, \tau) a_\alpha(x, \tau) - \frac{\lambda}{4} f^{\alpha\beta}(x, \tau) \int ds \Phi(\tau - s) f_{\alpha\beta}(x, s) \right] \quad (44)$$

leads to modified inhomogeneous pre-Maxwell equations

$$\partial_\beta \int ds \Phi(\tau - s) f^{\alpha\beta}(x, s) = e j^\alpha(x, \tau) \quad , \quad (45)$$

and by choosing $\Phi(\tau)$ as the inverse of $\varphi(\tau)$, such that

$$\int_{-\infty}^{\infty} ds \Phi(\tau - s) \varphi(s - r) = \delta(\tau - r) \quad , \quad (46)$$

it is possible to invert (46) to obtain the desired result

$$\partial_\beta \int d\tau' \varphi(\tau - \tau') \int ds \Phi(\tau' - s) f^{\alpha\beta}(x, s) = e \int d\tau' \varphi(\tau - \tau') j^\alpha(x, \tau') \quad (47)$$

$$\partial_\beta \int ds \delta(\tau - s) f^{\alpha\beta}(x, s) = e \int ds \varphi(\tau - s) j^\alpha(x, s) \quad (48)$$

$$\partial_\beta f^{\alpha\beta}(x, \tau) = e j_\varphi^\alpha(x, \tau) \quad . \quad (49)$$

For the Laplace distribution (33) used in [14] , the inverse function may be found readily in the “mass domain”. Expressing the distribution as a Fourier integral over “conjugate mass”,

$$\varphi(\tau) = \frac{1}{2\lambda} e^{-|\tau|/\lambda} = \int \frac{d\kappa}{2\pi} \tilde{\varphi}(\kappa) e^{-i\kappa\tau} \quad (50)$$

we find the mass regulation kernel

$$\tilde{\varphi}(\kappa) = \int d\tau e^{i\kappa\tau} \varphi(\tau) = \frac{1}{2\lambda} \int d\tau e^{i\kappa\tau} e^{-|\tau|/\lambda} = \frac{1}{1 + (\lambda\kappa)^2} \quad . \quad (51)$$

Similarly expressing the inverse function as a Fourier integral,

$$\Phi(\tau) = \int \frac{d\kappa}{2\pi} \tilde{\Phi}(\kappa) e^{-i\kappa\tau} \quad (52)$$

it follows that

$$\int_{-\infty}^{\infty} ds \Phi(\tau - s) \varphi(s - r) = \delta(\tau - r) \quad \Rightarrow \quad \tilde{\Phi}(\kappa) = \frac{1}{\tilde{\varphi}(\kappa)} = 1 + (\lambda\kappa)^2 \quad (53)$$

and so

$$\begin{aligned} \Phi(\tau) &= \int \frac{d\kappa}{2\pi} [1 + (\lambda\kappa)^2] e^{-i\kappa\tau} \\ &= \int \frac{d\kappa}{2\pi} e^{-i\kappa\tau} + \lambda^2 \int \frac{d\kappa}{2\pi} \kappa^2 e^{-i\kappa\tau} \\ &= \int \frac{d\kappa}{2\pi} e^{-i\kappa\tau} + \lambda^2 \int \frac{d\kappa}{2\pi} (i\partial_\tau)^2 e^{-i\kappa\tau} \\ &= \int \frac{d\kappa}{2\pi} e^{-i\kappa\tau} + \lambda^2 \left(i \frac{d}{d\tau} \right)^2 \int \frac{d\kappa}{2\pi} e^{-i\kappa\tau} \\ &= \delta(\tau) - \lambda^2 \delta''(\tau) . \end{aligned} \quad (54)$$

Inserting (54) into (43), the modified kinetic term for the gauge fields is

$$\begin{aligned} S_{em-kinetic} &= \frac{\lambda}{4} \int d^4x d\tau ds f^{\alpha\beta}(x, \tau) \Phi(\tau - s) f_{\alpha\beta}(x, s) \\ &= \frac{\lambda}{4} \int d^4x d\tau ds f^{\alpha\beta}(x, \tau) [\delta(\tau - s) - \lambda^2 \delta''(\tau - s)] f_{\alpha\beta}(x, s) \\ &= \frac{\lambda}{4} \int d^4x d\tau f^{\alpha\beta}(x, \tau) f_{\alpha\beta}(x, \tau) - \frac{\lambda^3}{4} \int d^4x d\tau f^{\alpha\beta}(x, \tau) \partial_\tau^2 f_{\alpha\beta}(x, \tau) \\ &= \frac{\lambda}{4} \int d^4x d\tau f^{\alpha\beta}(x, \tau) f_{\alpha\beta}(x, \tau) + \frac{\lambda^3}{4} \int d^4x d\tau (\partial_\tau f^{\alpha\beta}(x, \tau)) (\partial_\tau f_{\alpha\beta}(x, \tau)) \\ &= S_{em-kinetic}^0 + \frac{\lambda^3}{4} \int d^4x d\tau (\partial_\tau f^{\alpha\beta}(x, \tau)) (\partial_\tau f_{\alpha\beta}(x, \tau)) . \end{aligned} \quad (55)$$

In this form, the modified kinetic term appears as the usual term plus a higher-order derivative of the fields. Although the higher-order derivative breaks the formal $O(3,2)$ or $O(4,1)$ symmetry to $O(3,1)$, both Poincaré invariance and gauge invariance are preserved.

3 Quantization of the Interacting Theory

The modified action can be written in the form

$$S = \int d^4x d\tau \left\{ \psi^* (i\partial_\tau + e_0 a_5) \psi - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0 a_\mu) (-i\partial^\mu - e_0 a^\mu) \psi \right\} - \frac{\lambda}{4} \int d^4x d\tau ds f^{\alpha\beta}(x, \tau) \Phi(\tau - s) f_{\alpha\beta}(x, s) , \quad (56)$$

and we adopt the notation

$$(f^\Phi)_{\alpha\beta}(x, \tau) = \int ds \Phi(\tau - s) f_{\alpha\beta}(x, s) . \quad (57)$$

As shown in [11] and [16], a straight forward quantization is obtained by generalizing the Jackiw-Fadeev “first order quantization” method [17] to the path integral. Since the canonical structure of the matter field guarantees first-order form with respect to τ -derivatives, only the gauge field must be modified. Expanding the kinetic term

$$\begin{aligned} f^{\alpha\beta} (f^\Phi)_{\alpha\beta} &= f^{\mu\nu} (f^\Phi)_{\mu\nu} + 2f^{5\mu} (f^\Phi)_{5\mu} \\ &= f^{\mu\nu} (f^\Phi)_{\mu\nu} - 2f^{5\mu} (f^\Phi)_{\mu}^5 \\ &= f^{\mu\nu} (f^\Phi)_{\mu\nu} + 2 \left[2(\partial_\tau a^\mu + \partial^\mu a^5) (f^\Phi)_{\mu}^5 - f^{5\mu} (f^\Phi)_{\mu}^5 \right] , \end{aligned} \quad (58)$$

integrating by parts

$$(\partial_\tau a^\mu + \partial^\mu a^5) (f^\Phi)_{\mu}^5 = (\partial_\tau a^\mu) (f^\Phi)_{\mu}^5 - a^5 \partial^\mu (f^\Phi)_{\mu}^5 + \text{divergence} , \quad (59)$$

and introducing the definitions

$$\epsilon^\mu = f^{5\mu} \quad (\epsilon^\Phi)^\mu = (f^\Phi)_{\mu}^5 , \quad (60)$$

the action becomes

$$S = \int d^4x d\tau \left[i\psi^* \dot{\psi} - \lambda \dot{a}^\mu (\epsilon^\Phi)_\mu - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0 a_\mu) (-i\partial^\mu - e_0 a^\mu) \psi - \frac{\lambda}{4} f^{\mu\nu} (f^\Phi)_{\mu\nu} - \frac{\lambda}{2} \epsilon^\mu (\epsilon^\Phi)_\mu + a_5 \left(e_0 \psi^* \psi - \lambda \partial^\mu (\epsilon^\Phi)_\mu \right) \right] \quad (61)$$

where $\dot{\psi} = \partial_\tau \psi$ and $\dot{a}^\mu = \partial_\tau a^\mu$. Placing the action (61) into a path integral

$$\mathcal{Z} = \frac{1}{\mathcal{N}} \int \mathcal{D}\psi^* \mathcal{D}\psi \mathcal{D}a_\mu \mathcal{D}a_5 \mathcal{D}\epsilon_\mu e^{iS} \quad (62)$$

and noticing the absence of \dot{a}^5 , the integration over $\mathcal{D}a_5$ simply places

$$\delta \left(e_0 \psi^* \psi - \lambda \partial^\mu \left(\epsilon^\Phi \right)_\mu \right) \quad (63)$$

into the measure. The constraint (63) imposes the Gauss Law in the pre-Maxwell theory,

$$\partial^\mu \left(\epsilon^\Phi \right)_\mu = \frac{e_0}{\lambda} \psi^* \psi = e j^5 \quad (64)$$

and following Jackiw-Fadeev, we eliminate the constraint by solving (64). The solution may be found by a covariant decomposition into transverse and longitudinal parts

$$\left(\epsilon^\Phi \right)^\mu = \left(\epsilon_\perp^\Phi \right)^\mu + e \partial^\mu [G j^5] \quad (65)$$

where the transverse part satisfies

$$\partial_\mu \left(\epsilon_\perp^\Phi \right)^\mu = 0 \quad (66)$$

and the longitudinal part is found from

$$[G j^5](x, \tau) = \int d^4 y G(x - y) j^5(y, \tau) \quad (67)$$

with the Green's function

$$G(x - y) = \delta \left((x - y)^2 \right) \quad \Rightarrow \quad \square G = 1 . \quad (68)$$

Similarly, we impose the gauge fixing

$$\delta \left(\partial_\mu a^\mu - \Lambda \right) \quad (69)$$

which we solve through the decomposition

$$a^\mu = a_\perp^\mu + \partial^\mu [G \Lambda] \quad \partial_\mu a_\perp^\mu = 0 , \quad (70)$$

which further entails

$$\dot{a}^\mu = \dot{a}_\perp^\mu + \partial^\mu [G \dot{\Lambda}] \quad (71)$$

$$f^{\mu\nu} = \partial^\mu a_\perp^\nu - \partial^\nu a_\perp^\mu = f_\perp^{\mu\nu} \quad (72)$$

$$-i \partial^\mu - e_0 a^\mu = -i \partial^\mu - e_0 a_\perp^\mu - e_0 \partial^\mu [G \dot{\Lambda}] \quad (73)$$

$$\dot{a}^\mu (\epsilon^\Phi)_\mu = \dot{a}_\perp^\mu (\epsilon_\perp^\Phi)_\mu - e\rho[G\dot{\Lambda}] \quad . \quad (74)$$

Having eliminated the transverse components, the path integral is now

$$\mathcal{Z} = \frac{1}{\mathcal{N}} \int \mathcal{D}\psi^* \mathcal{D}\psi \mathcal{D}(a_\perp)_\mu \mathcal{D}(\epsilon_\perp)_\mu e^{iS} \quad (75)$$

where

$$\begin{aligned} S = & \int d^4x d\tau \left[i\psi^* \dot{\psi} - \lambda \dot{a}_\perp^\mu (\epsilon_\perp^\Phi)_\mu + \lambda e j^5 [G\dot{\Lambda}] \right. \\ & - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0 a_{\perp\mu} - e_0 \partial_\mu [G\dot{\Lambda}]) (-i\partial^\mu - e_0 a_\perp^\mu - e_0 \partial^\mu [G\dot{\Lambda}]) \psi \\ & \left. - \frac{\lambda}{4} f_\perp^{\mu\nu} (f_\perp^\Phi)_{\mu\nu} - \frac{\lambda}{2} \epsilon_\perp^\mu (\epsilon_\perp^\Phi)_\mu + \frac{\lambda}{2} e^2 j^5 [Gj^5] \right] \quad . \quad (76) \end{aligned}$$

The next step is to perform the Gaussian integration over $\mathcal{D}(\epsilon_\perp)_\mu$

$$\begin{aligned} & \int \mathcal{D}\epsilon_{\perp\mu} \exp \left[-i \int d^4x d\tau \frac{\lambda}{2} \epsilon_\perp^\mu (\epsilon_\perp^\Phi)_\mu + \lambda \dot{a}_\perp^\mu (\epsilon_\perp^\Phi)_\mu \right] = \\ & = \int \mathcal{D}\epsilon_{\perp\mu} \exp \left[-i \int d^4x d\tau ds \frac{\lambda}{2} \epsilon_\perp^\mu \Phi \epsilon_{\perp\mu} + \lambda \dot{a}_\perp^\mu \Phi \epsilon_{\perp\mu} \right] \\ & = \exp \left[-i \frac{\lambda}{2} \int d^4x d\tau ds \dot{a}_\perp^\mu \Phi \dot{a}_{\perp\mu} \right] \\ & = \exp \left[-i \frac{\lambda}{2} \int d^4x d\tau \dot{a}_\perp^\mu (\dot{a}_\perp^\Phi)_\mu \right] \quad . \quad (77) \end{aligned}$$

The path integral now takes the form

$$\mathcal{Z} = \frac{1}{\mathcal{N}} \int \mathcal{D}\psi^* \mathcal{D}\psi \mathcal{D}(a_\perp)_\mu e^{iS} \quad (78)$$

where

$$\begin{aligned} S = & \int d^4x d\tau \left[i\psi^* \dot{\psi} + \lambda e j^5 [G\dot{\Lambda}] \right. \\ & - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0 a_{\perp\mu} - e_0 \partial_\mu [G\dot{\Lambda}]) (-i\partial^\mu - e_0 a_\perp^\mu - e_0 \partial^\mu [G\dot{\Lambda}]) \psi \\ & \left. - \frac{\lambda}{4} f_\perp^{\mu\nu} (f_\perp^\Phi)_{\mu\nu} + \frac{\lambda}{2} \dot{a}_\perp^\mu (\dot{a}_\perp^\Phi)_\mu + \frac{\lambda}{2} e^2 j^5 [Gj^5] \right] \quad , \quad (79) \end{aligned}$$

and eliminating the gauge fixing with the transformation

$$\psi \longrightarrow e^{ie_0[G\dot{\Lambda}]} \psi \quad (80)$$

we are left with

$$\mathcal{Z} = \frac{1}{\mathcal{N}} \int \mathcal{D}\psi^* \mathcal{D}\psi \mathcal{D}(a_\perp)_\mu e^{iS} \quad (81)$$

where the action has the unconstrained transverse form

$$\begin{aligned} S = \int d^4x d\tau \left\{ i\psi^* \dot{\psi} - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0(a_\perp)_\mu) (-i\partial^\mu - e_0(a_\perp)^\mu) \psi \right. \\ \left. - \frac{\lambda}{4} (f_\perp)^{\mu\nu} (f_\perp^\Phi)_{\mu\nu} + \frac{\lambda}{2} (\dot{a}_\perp)_\mu (\dot{a}_\perp^\Phi)^\mu + \frac{\lambda}{2} e^2 j^5 [Gj^5] \right\} . \end{aligned} \quad (82)$$

Finally, making the rearrangements

$$\begin{aligned} -\frac{\lambda}{4} (f_\perp)^{\mu\nu} (f_\perp^\Phi)_{\mu\nu} &= -\frac{\lambda}{4} [\partial^\mu a_\perp^\nu - \partial^\nu a_\perp^\mu] [\partial_\mu (a_\perp^\Phi)_\nu - \partial_\nu (a_\perp^\Phi)_\mu] \\ &= -\frac{\lambda}{2} [(\partial^\mu a_\perp^\nu) (\partial_\mu (a_\perp^\Phi)_\nu) - (\partial^\nu a_\perp^\mu) (\partial_\nu (a_\perp^\Phi)_\mu)] \\ &= \frac{\lambda}{2} a_\mu [g^{\mu\nu} \square - \partial^\mu \partial^\nu] (a_\perp^\Phi)_\nu + \text{divergence} \end{aligned} \quad (83)$$

and

$$(\dot{a}_\perp)_\mu (\dot{a}_\perp^\Phi)^\mu = -(a_\perp)_\mu [g^{\mu\nu} \partial_\tau^2] (a_\perp^\Phi)_\nu + \text{divergence} \quad (84)$$

the action becomes

$$\begin{aligned} S = \int d^4x d\tau \left\{ i\psi^* \dot{\psi} - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0(a_\perp)_\mu) (-i\partial^\mu - e_0(a_\perp)^\mu) \psi \right. \\ \left. + \frac{\lambda}{2} (a_\perp)_\mu [\square + \sigma \partial_\tau^2] (a_\perp^\Phi)^\mu + \frac{\lambda}{2} e^2 j^5 [Gj^5] \right\} . \end{aligned} \quad (85)$$

Inserting the explicit form for the inverse Laplace distribution $\Phi(\tau)$

$$\begin{aligned} S = \int d^4x d\tau \left\{ i\psi^* \dot{\psi} - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0(a_\perp)_\mu) (-i\partial^\mu - e_0(a_\perp)^\mu) \psi \right. \\ \left. + \frac{\lambda}{2} e^2 j^5 [Gj^5] \right\} + \int d^4x d\tau ds \frac{\lambda}{2} (a_\perp)^\mu [\square + \sigma \partial_\tau^2] \Phi a_{\perp\mu} \end{aligned} \quad (86)$$

$$\begin{aligned} S = \int d^4x d\tau \left\{ i\psi^* \dot{\psi} - \frac{1}{2M} \psi^* (-i\partial_\mu - e_0(a_\perp)_\mu) (-i\partial^\mu - e_0(a_\perp)^\mu) \psi \right. \\ \left. + \frac{\lambda}{2} (a_\perp)^\mu [\square + \sigma \partial_\tau^2] (1 - \lambda^2 \partial_\tau^2) a_{\perp\mu} + \frac{\lambda}{2} e^2 j^5 [Gj^5] \right\} . \end{aligned} \quad (87)$$

we can now read the Feynman rules from the action (87). We may summarize the Feynman rules for the momentum space Green's functions as follows:

1. For each matter field propagator, draw a directed line associated with the factor

$$\frac{1}{(2\pi)^5} \frac{-i}{\frac{1}{2M}p^2 - P - i\epsilon} \quad (88)$$

2. For each photon propagator, draw a photon line associated with the factor

$$\frac{1}{\lambda} \left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right] \frac{-i}{k^2 + \kappa^2 - i\epsilon} \frac{1}{1 + \lambda^2 \kappa^2} \quad (89)$$

3. For the three-particle interaction, write the vertex factor

$$\frac{e_0}{2M} i(p + p')^\nu (2\pi)^5 \delta^4(p - p' - k) \delta(P - P' - \kappa) \quad (90)$$

4. For the four-particle interaction, write the vertex factor

$$\frac{-ie_0^2}{M} (2\pi)^5 g_{\mu\nu} \delta^4(k - k' - p' + p) \delta(-\kappa + \kappa' + P' - P) \quad (91)$$

These Feynman rules are identical to those given in [11] and [16], with the addition of the mass regulating factor $\frac{1}{1+\lambda^2\kappa^2}$ in (89). It was shown in [11] that with an appropriate mass cut-off, the off-shell quantum electrodynamics can be counter-term renormalized. In fact, since the matter propagator is retarded, in the sense that

$$G(x - x', \tau - \tau') = 0, \quad \tau < \tau' \quad , \quad (92)$$

there are no matter loops in the resulting theory, and the mass-regulated theory is super-renormalizable. With the mass regulation offered by the higher-order kinetic term, pre-Maxwell quantum electrodynamics is essentially finite — only the one-photon loop self-energy diagram diverges as

$$G_0^{(2)}(p) \left((2\pi)^5 \frac{ie_0^2}{M} \right)^2 \frac{1}{\lambda} \int d^4q dQ \frac{-i}{q^2 + Q^2 - i\epsilon} \frac{1}{1 + \lambda^2 Q^2} G_0^{(2)}(p) \quad (93)$$

and this diagram is renormalized by shifting the term $i\psi^* \partial_\tau \psi$. The problematic divergent diagram in the unregulated theory is given by the overlapping two-photon diagram, and contributes the following term to the matter propagator:

$$\begin{aligned}
& G_0^{(2)}(p) G_0^{(2)}(p) \left((2\pi)^5 \frac{ie_0}{2M} \right)^4 16p^4 \int d^4q dQ d^4q' dQ' \\
& \frac{1}{\lambda^2} \frac{1}{q^2 - Q^2} \frac{1}{q'^2 - Q'^2} \frac{1}{(p-q)^2 - 2M(P-Q)} \frac{2M}{2M} \\
& \frac{2M}{(p-q')^2 - 2M(P-Q')} \frac{2M}{(p-q-q')^2 - 2M[P - (Q+Q')]} \quad . \quad (94)
\end{aligned}$$

Since this term includes the factor p^4 , it cannot be renormalized by a counter-term, and a mass cut-off is required. With the regulation provided by the higher-order kinetic term, this contribution is now completely finite:

$$\begin{aligned}
& G_0^{(2)}(p) G_0^{(2)}(p) \left((2\pi)^5 \frac{ie_0}{2M} \right)^4 16p^4 \int d^4q dQ d^4q' dQ' \\
& \frac{1}{\lambda^2} \frac{1}{q^2 - Q^2} \left\{ \frac{1}{1 - \lambda^2 Q^2} \right\} \frac{1}{q'^2 - Q'^2} \left\{ \frac{1}{1 - \lambda^2 Q'^2} \right\} \frac{2M}{(p-q)^2 - 2M(P-Q)} \\
& \frac{2M}{(p-q')^2 - 2M(P-Q')} \frac{2M}{(p-q-q')^2 - 2M[P - (Q+Q')]} \quad . \quad (95)
\end{aligned}$$

4 Conclusion

We have shown that the addition of the higher-order kinetic term for the gauge fields provides a Lorentz-covariant and gauge invariant approach to both mass regulation in pre-Maxwell quantum electrodynamics, and to a corresponding smoothing of the classical electromagnetic potential, leading to a reasonable description of Coulomb scattering. It is particularly interesting that this modification of the kinetic term for the gauge fields is equivalent to the introduction of parameterization uncertainty into the particle-field interaction. Moreover, the appearance of the Laplace distribution — hinting at an underlying Poisson process assigning events along particle world lines — suggests further directions of exploration in pre-Maxwell theory and higher-order Lagrangians.

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